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Maxwell's equation of electromagnetism in free space

Maxwell's equations of electromagnetism are

$$\vec{\nabla}\cdot\vec{D} = \rho$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\delta\vec{B}}{\delta t}$$
$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\delta\vec{D}}{\delta t}$$

For free space, $\rho = 0$ and $J = 0$
so Maxwell's equations of electromagnetism in free space will be

$$\vec{\nabla}\cdot\vec{D} = 0 \quad \text{--- (i)}$$
$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{--- (ii)}$$
$$\vec{\nabla}\times\vec{E} = -\frac{\delta\vec{B}}{\delta t} \quad \text{--- (iii)}$$
$$\vec{\nabla}\times\vec{H} = \frac{\delta\vec{D}}{\delta t} \quad \text{--- (iv)}$$

Wave equations for $\vec{E}$ and $\vec{H}$ in free space

* Wave equations for $\vec{E}$ in free space is $\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\delta^2\vec{E}}{\delta t^2}$
* Wave equations for $\vec{B}$ in free space is $\nabla^2\vec{B} = \mu_0\epsilon_0\frac{\delta^2\vec{B}}{\delta t^2}$.

Derivation of $\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\delta^2\vec{E}}{\delta t^2}$ in free space

Maxwell's ~~field~~ equations of electromagnetism in free space are

$$\vec{\nabla}\cdot\vec{D} = 0 \quad \text{--- (i)} \qquad \vec{\nabla}\cdot\vec{H} = 0 \quad \text{--- (ii)}$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{--- (iii)} \qquad \vec{\nabla}\times\vec{H} = \frac{\delta\vec{D}}{\partial t} \quad \text{--- (iv)}$$

From eqn (iii), $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

Taking curl both sides, we get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$
$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) \quad \text{--- (1)}$$

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Now $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{E}(\vec{\nabla} \cdot \vec{\nabla})$

$\{\because \vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})\}$

$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \frac{\vec{D}}{\varepsilon_0}) - \vec{E}\,\nabla^2 \quad \because \vec{E} = \frac{\vec{D}}{\varepsilon_0},\ \vec{\nabla} \cdot \vec{\nabla} = \nabla^2$

$= \frac{1}{\varepsilon_0} \vec{\nabla}(\vec{\nabla} \cdot \vec{D}) - \nabla^2 \vec{E}$

$= \frac{1}{\varepsilon_0} \vec{\nabla} \cdot 0 - \nabla^2 \vec{E} \quad \because \vec{\nabla} \cdot \vec{D} = 0$ From equ (i)

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\nabla^2 \vec{E} \qquad (2)$$

Now $\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}) = \frac{\partial}{\partial t}(\vec{\nabla} \times \mu_0 \vec{H}) \quad \because \vec{B} = \mu_0 \vec{H}$

$= \mu_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H})$

$= \mu_0 \frac{\delta}{\delta t}\left(\frac{\delta \vec{D}}{\delta t}\right) \quad \because \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ From equ (iv)

$= \mu_0 \frac{\delta^2}{\delta t^2}(\varepsilon_0 \vec{E}) \quad \because \vec{D} = \epsilon_0 \vec{E}$

$$\frac{\partial}{\delta t}(\vec{\nabla} \times \vec{B}) = \mu_0 \varepsilon_0 \frac{\delta^2 \vec{E}}{\delta t^2} \qquad (3)$$

Using equs (2) and (3) in equ (1), we get

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\delta^2 \vec{E}}{\delta t^2}$$

$$\Rightarrow \boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\delta^2 \vec{E}}{\delta t^2}}$$

It is wave equation ~~for E~~ in free space in terms of $\vec{E}$.

In component form

$$\frac{\delta^2 E_x}{\delta x^2} = \mu_0 \varepsilon_0 \frac{\delta^2 E_x}{\delta t^2}$$

$$\frac{\delta^2 E_y}{\delta y^2} = \mu_0 \varepsilon_0 \frac{\delta^2 E_y}{\delta t^2}$$

and $$\frac{\delta^2 E_z}{\delta z^2} = \mu_0 \varepsilon_0 \frac{\delta^2 E_z}{\delta t^2}$$

These are wave equation ~~for E~~ in free space in three mutually perpendicular components of $\vec{E}$.

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Derivation of wave equation $\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\delta^2 \vec{B}}{\delta t^2}$ in free space

Maxwell's equation of electromagnetism in free space is

$$\vec{\nabla}\cdot\vec{D} = 0 \text{ —— (i)} \qquad \vec{\nabla}\cdot\vec{H} = 0 \text{ —— (ii)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\delta \vec{B}}{\delta t} \text{ —— (iii)} \qquad \vec{\nabla}\times\vec{H} = \frac{\delta \vec{D}}{\delta t} \text{ —— (iv)}$$

From equⁿ (iv), $\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$

Taking curl both sides, We get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\times\frac{\delta\vec{D}}{\delta t}$$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \frac{\delta}{\delta t}(\vec{\nabla}\times\vec{D}) \text{ —— (1)}$$

Now $\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \vec{H}(\vec{\nabla}\cdot\vec{\nabla})$

$\{\because \vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})\}$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}.0 - \vec{H}.\nabla^2 \qquad \because \vec{\nabla}\cdot\vec{H} = 0 \text{ From equ}^n\text{ (ii) and } \vec{\nabla}\cdot\vec{\nabla} = \nabla^2$$

$$= -\frac{\vec{B}}{\mu_0}.\nabla^2$$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = -\frac{1}{\mu_0}.\nabla^2\vec{B} \text{ —— (2)}$$

Now $\frac{\delta}{\delta t}(\vec{\nabla}\times\vec{D}) = \frac{\partial}{\partial t}(\vec{\nabla}\times\varepsilon_0\vec{E}) \qquad \because \vec{D} = \varepsilon_0\vec{E}$

$$= \varepsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

$$= \varepsilon_0\frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right) \qquad \because \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \text{ From equ}^n\text{ (iii)}$$

$$\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{D}) = -\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} \text{ —— (3)}$$

Using equ^ns (2) and (3) in equ^n (1), we get

$$-\frac{1}{\mu_0}\cdot\nabla^2\vec{B} = -\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}}$$

It is wave equation ~~[illegible]~~ in free space in terms of $\vec{B}$

In component form:

$$\frac{\partial^2 B_x}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B_x}{\partial t^2}$$

$$\frac{\partial^2 B_y}{\partial y^2} = \mu_0\varepsilon_0\frac{\partial^2 B_y}{\partial t^2}$$

and $$\frac{\partial^2 B_z}{\partial z^2} = \mu_0\varepsilon_0\frac{\partial^2 B_z}{\partial t^2}$$

These are wave equation ~~[illegible]~~ in free space in three mutually perpendicular components of $\vec{B}$.

Determination of Electromagnetic waves velocity

EM wave equation in free space in terms of $\vec{E}$ is

$$\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \quad \text{——— (1)}$$

Wave equation in general form is

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{——— (2)}$$

Comparing equ^ns (1) and (2) we get

$$\frac{1}{v^2} = \mu_0\varepsilon_0 \Rightarrow v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = \text{velocity of light in free space}$$

$\mu_0 = 4\pi\times10^{-7}\,Tm/A$ and $\varepsilon_0 = \frac{1}{36\pi\times10^{9}}\,C^2/Nm^2$

$$v = \frac{1}{\sqrt{4\pi\times10^{-7}\times\frac{1}{36\pi\times10^9}}} = \frac{1}{\sqrt{\frac{1}{9\times10^{16}}}} = \sqrt{9\times10^{16}} = 3\times10^8\,m/s.$$